

Determining Neutrino Mass Hierarchy by Precision Measurements in Electron and Muon Neutrino Disappearance Experiments

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Abstract

Recently a new method for determining the neutrino mass hierarchy by comparing the effective values of the atmospheric Δm^2 measured in the electron neutrino disappearance channel, $\Delta m^2(\text{ee})$, with the one measured in the muon neutrino disappearance channel, $\Delta m^2(\mu\mu)$, was proposed. If $\Delta m^2(\text{ee})$ is larger (smaller) than $\Delta m^2(\mu\mu)$ the hierarchy is of the normal (inverted) type. We re-examine this proposition in the light of two very high precision measurements: $\Delta m^2(\mu\mu)$ that may be accomplished by the phase II of the Tokai-to-Kamioka (T2K) experiment, for example, and $\Delta m^2(\text{ee})$ that can be envisaged using the novel Mössbauer enhanced resonant $\bar{\nu}_e$ absorption technique. Under optimistic assumptions for the systematic uncertainties of both measurements, we estimate the parameter region of (θ_{13}, δ) in which the mass hierarchy can be determined. If θ_{13} is relatively large, $\sin^2 2\theta_{13} \gtrsim 0.05$, and both of $\Delta m^2(\text{ee})$ and $\Delta m^2(\mu\mu)$ can be measured with the precision of $\sim 0.5\%$ it is possible to determine the neutrino mass hierarchy at $> 95\%$ CL for $0.3\pi \lesssim \delta \lesssim 1.7\pi$ for the current best fit values of all the other oscillation parameters.

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In spite of the great progress that has been made in recent years there still remains a number of important questions regarding the nature of neutrinos. One such unknown is whether the neutrinos have a normal or inverted mass hierarchy. This unresolved ambiguity in the neutrino mass pattern is usually phrased as follows: if the neutrino masses are labeled as m_1 (m_3) for the mass of the neutrino state with the greatest (least) ν_e component then for the normal hierarchy $m_3 > m_2 > m_1$ and for the inverted hierarchy $m_2 > m_1 > m_3$. The determination of this pattern must shed light on the secret of how the lepton sector is organized and may testify to the underlying symmetries by which the structure of neutrino masses and lepton flavor mixing are prescribed [1].

It is widely recognized that it is difficult to determine the neutrino mass hierarchy by any of the ongoing and the near future neutrino oscillation experiments including the atmospheric neutrino observation by Super-Kamiokande [2], MINOS [3], OPERA [4], and T2K (Tokai-to-Kamioka) [5]. The most studied method for determining the mass hierarchy within the scope of the future neutrino experiments is to explore the earth matter effect in long-baseline accelerator experiments [6]. Since the way the matter effect interferes with vacuum oscillation depends upon the sign of $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$, the degeneracy between the normal and the inverted mass hierarchies can be lifted in matter. The NO ν A [7] and the T2KK (Tokai-to-Kamioka-Korea) [8] experiments are designed to have a significant matter effect so that the hierarchy may be determined under favorable values of the unknown neutrino parameters. Although it is generally accepted that long-baseline experiments are the most promising method for determining the mass hierarchy, alternative ways should be explored especially if they have the most sensitivity for neutrino parameters which are complementary to the favorable parameters for long baseline experiments.

It was proposed in [9, 10] that accurate measurement of effective atmospheric scale Δm^2 both in electron and muon neutrino disappearance channels would determine the neutrino mass hierarchy, provided that precision of the measurement could reach sub percent level. We call this the “ Δm^2 Disappearance” method. In this paper, we describe a setting by which such precision measurement could be achieved, and perform a semi-quantitative analysis to estimate the parameter region in which the mass hierarchy can be determined by this method. For the measurement of Δm^2 in ν_μ disappearance channel, which will be denoted as $\Delta m^2(\mu\mu)$, we assume a factor of 2-4 improvement with respect to what is currently expected for the phase II of the T2K project. For the measurement of $\Delta m^2(ee)$, we assume an experiment based on the resonant absorption of $\bar{\nu}_e$ enhanced by the Mössbauer effect as recently proposed by Raghavan [11].

After reviewing the “ Δm^2 Disappearance” method in Sec. I, we discuss the accuracy of the determination of $\Delta m^2(\mu\mu)$ in Sec. II, and recollect the one for $\Delta m^2(ee)$ in Sec. III. In Sec. IV we present our analysis method and results. Finally, in Sec. V we draw our conclusions.

I. DETERMINING THE NEUTRINO MASS HIERARCHY BY ν_e AND ν_μ DISAPPEARANCE MEASUREMENTS - “ Δm^2 DISAPPEARANCE” METHOD

Let us first explain the unconventional way of determining neutrino mass hierarchy, the “ Δm^2 Disappearance” method [9, 10], to be explored in this paper. Suppose that we measure the atmospheric Δm^2 by doing disappearance measurements of $\nu_e \rightarrow \nu_e/\bar{\nu}_e \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_\mu/\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ in vacuum. For clarity, we start with a simplified setting where the solar mixing angle θ_{12} vanishes. In this case, it is obvious that $\nu_e \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_\mu$ channels are

governed by Δm_{31}^2 and approximately Δm_{32}^2 , respectively, since the two oscillations scales approximately decouple from each other. Notice that because $m_2 > m_1$, $|\Delta m_{31}^2| > |\Delta m_{32}^2|$ for the normal hierarchy and $|\Delta m_{31}^2| < |\Delta m_{32}^2|$ for the inverted hierarchy. Therefore, in the hypothetical simplified world of vanishing θ_{12} , one could, in principle, determine the mass hierarchy just by comparing the absolute values of the two Δm^2 , $|\Delta m_{31}^2| = \Delta m^2(\text{ee})$ and $|\Delta m_{32}^2| \approx \Delta m^2(\mu\mu)$.

For non-zero θ_{12} , the same considerations apply except that the difference is reduced by approximately $\cos 2\theta_{12}$. To make the discussion transparent we introduce, following [9], the effective atmospheric $\Delta m^2(\alpha\alpha)$ determined by the disappearance measurement in the $\nu_\alpha \rightarrow \nu_\alpha$ channel. It is this effective $\Delta m^2(\alpha\alpha)$, obtained by a two-flavor approximate description of the full three-flavor situation, which contains to high accuracy all the three-flavor mixing effects. Let us briefly review the derivation of $\Delta m^2(\alpha\alpha)$. In the approximate two-flavor mixing framework, the survival probability is given by

$$1 - P(\nu_\alpha \rightarrow \nu_\alpha) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2(\alpha\alpha) L}{4 E} \right). \quad (1)$$

The energy $E_{\text{dip}}^{2\text{-flavor}}$ at which $1 - P(\nu_\alpha \rightarrow \nu_\alpha)$ is maximum is given by $\Delta m^2(\alpha\alpha)L = 2\pi E_{\text{dip}}^{2\text{-flavor}}$. Now we repeat the same computation with the full three-flavor expression of the survival probability; $E_{\text{dip}}^{3\text{-flavor}}$ can be obtained by solving $d[1 - P(\nu_\alpha \rightarrow \nu_\alpha)]/dE = 0$. By demanding that $E_{\text{dip}}^{2\text{-flavor}} = E_{\text{dip}}^{3\text{-flavor}}$, an expression for $\Delta m^2(\alpha\alpha)$ can be derived. Assuming vacuum oscillations and ignoring the terms of the order of $\sim (\Delta m_{21}^2/\Delta m_{31}^2)^2$, we obtain [9],

$$\Delta m^2(\alpha\alpha) = r_\alpha |\Delta m_{31}^2| + (1 - r_\alpha) |\Delta m_{32}^2|, \quad (2)$$

where

$$r_\alpha \equiv \frac{|U_{\alpha 1}|^2}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2}. \quad (3)$$

The $U_{\alpha i}$'s are elements of the neutrino mixing matrix [12], for which we use the standard parametrization [13]. The physical meaning of r_α is that it is the fraction of the α flavor in the ν_1 eigenstate over the sum of this fraction in the ν_1 and ν_2 eigenstates. Note that $r_e = \cos^2 \theta_{12}$ without further approximation and $r_\mu \approx \sin^2 \theta_{12}$ if $\sin \theta_{13} \ll 1$.

The difference $\Delta_{e\mu}$ of the two effective Δm^2 can be easily computed from Eq. (2) and (3) to be

$$\Delta_{e\mu} \equiv \Delta m^2(\text{ee}) - \Delta m^2(\mu\mu) = (r_e - r_\mu)(|\Delta m_{31}^2| - |\Delta m_{32}^2|). \quad (4)$$

Where

$$r_e - r_\mu = \cos 2\theta_{12} - \cos \delta \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23} + \mathcal{O}(\sin^2 \theta_{13}), \quad (5)$$

which is positive for small values of $\sin \theta_{13}$, and

$$|\Delta m_{31}^2| - |\Delta m_{32}^2| = \pm \Delta m_{21}^2, \quad (6)$$

where the $+$ ($-$) sign is for the normal (inverted) hierarchy.¹ Thus the *sign* of $\Delta_{e\mu}$ is the same as the *sign* of Δm_{31}^2 . Therefore, within the experimentally allowed region for the mixing parameters², the hierarchy is normal if $\Delta_{e\mu}$ is positive and inverted if $\Delta_{e\mu}$ is negative.

¹ SNO's demonstration [14] that the solar CC/NC ratio is less than $\frac{1}{2}$ implies $\Delta m_{21}^2 > 0$.

² Except for some extreme values.

One should be reminded that *both* measurements, $\Delta m^2(\text{ee})$ and $\Delta m^2(\mu\mu)$, have to be accurate to the 1% level at least, because $|\Delta_{\text{e}\mu}/\Delta m^2(\alpha\alpha)| \sim 1\%$ for the current best fitted parameters. A more elaborate treatment in [9] entailed a rough estimation that both accuracies must be better than 0.5%. Therefore, the issue is whether one can at least envisage experimental setups capable of measuring both Δm^2 to such a high accuracy.

II. PRECISION MEASUREMENT OF $\Delta m^2(\mu\mu)$ BY T2K II EXPERIMENT

It appears that precision determination of $\Delta m^2(\mu\mu)$ is more feasible because there exists a well known method: the spectrum measurement in the ν_μ disappearance channel. If it can be performed with high precision, it should be possible to determine $\Delta m^2(\mu\mu)$ with great accuracy.

Unfortunately, there is a potential obstacle to accurate measurement of $\Delta m^2(\mu\mu)$, the problem of *absolute energy scale uncertainty*. For concreteness, we consider the phase II of the T2K project (T2K II) [5] where the beam power from J-PARC is upgraded to 4 MW and the far detector will be Hyper-Kamiokande with 0.54 Mt fiducial volume. With huge number of events obtained in several years of running, the precision of $\Delta m^2(\mu\mu)$ would reach to $\sim 0.1\%$ level without the energy scale uncertainty [15]. To determine Δm^2 accurately, however, we have to know precisely the absolute value of the energy of the detected muons to reconstruct the neutrino energies. Since the oscillation probability depends on $\Delta m^2/E$, the accuracy of $\Delta m^2(\mu\mu)$ measurement is limited by the energy scale error. At the moment, the absolute energy of muons are known to the accuracy of 2% at energies ~ 1 GeV in the Super-Kamiokande detector [2]. Lacking better calibration sources, one expects that the energy scale uncertainty will be the major limiting factor in improving the $\Delta m^2(\mu\mu)$ determination in the T2K II experiment [16]. Notice that the difficulty is not specific to this experiment and similar statements would apply to other detection methods as well.

If the 2% energy scale uncertainty still remains, our method for determining the mass hierarchy would not work because then the maximum precision we can expect on $\Delta m^2(\mu\mu)$ is 2%. See Appendix A. A precision better than 1% is needed for the method discussed in this paper to be useful. In this work, we take the optimistic attitude that this uncertainty can be reduced to the level of 0.5%-1% by the time T2K II will be realized, or by development of alternative detection technologies. We study to what extent one can determine the mass hierarchy with the method described in the previous section. We believe that physics results we can achieve under such an assumption are worthwhile to report.

For definiteness, we assume that $\Delta m^2(\mu\mu)$ can be determined with 0.5% accuracy. We also examine the case in which the accuracy is 1% to reveal how the capability of the mass hierarchy determination depends upon the accuracy of $\Delta m^2(\mu\mu)$. We note that due to the fact that a large oscillation effect in the $\nu_\mu \rightarrow \nu_\mu$ mode is expected, the precision in the determination of $\Delta m^2(\mu\mu)$ essentially does not depend on the precise values of the unknown mixing parameters such as θ_{13} and δ , and possible deviation of θ_{23} from $\pi/4$. Therefore, the fractional accuracy, $\delta(\Delta m^2(\mu\mu))/\Delta m^2(\mu\mu)$ can be regarded, to good approximation, as constant for a given systematic uncertainty. Also, since matter effects change the size of $\Delta m^2(\mu\mu)$ by less than 0.1% they can be ignored for the T2K measurement of $\Delta m^2(\mu\mu)$, see [9].

III. PRECISION MEASUREMENT OF $\Delta m^2(\text{ee})$ BY MÖSSBAUER ENHANCED RESONANT ABSORPTION OF $\bar{\nu}_e$

Most probably, the most difficult part of the method for the determination of the neutrino mass hierarchy explored in this paper is to measure $\Delta m^2(\text{ee})$ to the level of a few parts per mil. The reactor θ_{13} experiments [17], as they stand, will not reach the accuracy of 1% level. Recently, the intriguing possibility of using the resonant absorption reaction [18]

$$\bar{\nu}_e + {}^3\text{He} + \text{orbital } e^- \rightarrow {}^3\text{H}, \quad (7)$$

to explore $\bar{\nu}_e$ interaction below the threshold of the charged-current absorption reaction $\bar{\nu}_e + p \rightarrow n + e^+$, has been proposed by Raghavan [11]. See Ref. [19, 20] for earlier suggestions. The resonance condition is automatically satisfied if the $\bar{\nu}_e$ beam is prepared with the use of the inverse reaction ${}^3\text{H} \rightarrow \bar{\nu}_e + {}^3\text{He} + \text{orbital } e^-$. If both, the source and the target, atoms are placed in a metal lattice and can enjoy the same environment the resonant cross section can be enhanced by the Mössbauer effect by a factor of $\sim 10^{11}$ [11]. See also [21] where some potential difficulties of this experiment are discussed.

Because the energy of $\bar{\nu}_e$ from the bound state beta decay is so low, $E = 18.6$ keV, the first oscillation maximum (minimum of $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$) is reached at the baseline distance $L_{\text{OM}} = 9.2 \times (|\Delta m_{31}^2|/2.5 \times 10^{-3} \text{eV}^2)^{-1}$ m. Then, one can envisage θ_{13} experiments with ~ 10 m baseline [11]. With the Mössbauer enhanced cross section of $\sigma_{\text{res}} \simeq 5 \times 10^{-32} \text{cm}^2$ the rate $R \equiv N_T f_{\bar{\nu}_e} \sigma_{\text{res}}$, with N_T being the number of target atoms and $f_{\bar{\nu}_e}$ the neutrino flux, is given by

$$R_{\text{enh}} = 1.2 \times 10^6 \left(\frac{S M_T}{1 \text{ MCi} \cdot 100 \text{ g}} \right) \left(\frac{L}{10 \text{ m}} \right)^{-2} \text{day}^{-1}, \quad (8)$$

that is, a million events a day by using 100 g of ${}^3\text{He}$ target mass M_T and assuming a source strength S of 1 MCi.

It is natural to expect that the monochromatic nature of the $\bar{\nu}_e$ beam as well as the extremely high statistics allow one to measure $\Delta m^2(\text{ee})$ to a high accuracy, as demonstrated by the recent analysis in [22]. For the purpose of the present analysis, we use one of the particular settings discussed in that paper, the one called Run IIB, which is defined as follows:

Run IIB: it consists of measurements at 10 different detector locations; $L_n = (2n+1) L_{\text{OM}}/5$ for $n = 1, \dots, 10$, so that the entire period, from 0 to 2π , is covered. At each location an equal number of 10^6 events is to be collected.

The authors of [22] argued that if the direct counting of produced ${}^3\text{H}$ atoms works, a movable detector technique would allow for a relative systematic uncertainty as low as 0.2%, and if not it may be of the order of $\simeq 1\%$. If the former uncertainty is realized, a sensitivity to $\Delta m^2(\text{ee})$ of $\simeq 0.3 (\sin^2 2\theta_{13}/0.1)^{-1}\%$ at 1σ CL is possible. For the latter uncertainty, the sensitivity is worse by about a factor of four [22]. In this work, we consider this systematic uncertainty to be 0.2%, and also discuss the case where it is 1%.

IV. ANALYSIS METHOD AND RESULTS

Given the accuracies of the measurements of $\Delta m^2(\mu\mu)$ and $\Delta m^2(\text{ee})$ discussed in Secs. II and III, it is straightforward to determine the region of mixing parameters in which the

mass hierarchy can be resolved. To understand what are the relevant parameters, we show in Fig. 1 the 1σ CL determination of $\Delta m^2(\text{ee})$ and $\Delta m^2(\mu\mu)$ as a function of $\sin^2 2\theta_{13}$ for $\cos\delta = -1$ (left panel), 0 (middle panel) and 1 (right panel). The physical parameters, Δm_{31}^2 and Δm_{32}^2 , are held fixed, which implies that $\Delta m^2(\text{ee})$ is also fixed. If the two strips of $\Delta m^2(\text{ee})$ and $\Delta m^2(\mu\mu)$ do not overlap then the mass hierarchy is determined at better than 84% CL. Even though they overlap the mass hierarchy can be determined to the extent that one can discriminate if $\Delta_{e\mu} \equiv \Delta m^2(\text{ee}) - \Delta m^2(\mu\mu)$ is positive (normal hierarchy) or negative (inverted hierarchy). Throughout this section we use the following values for the solar oscillation parameters: $\Delta m_{21}^2 = 8.0 \times 10^{-5} \text{eV}^2$ and $\sin^2 \theta_{12} = 0.31$ [14], unless stated otherwise.

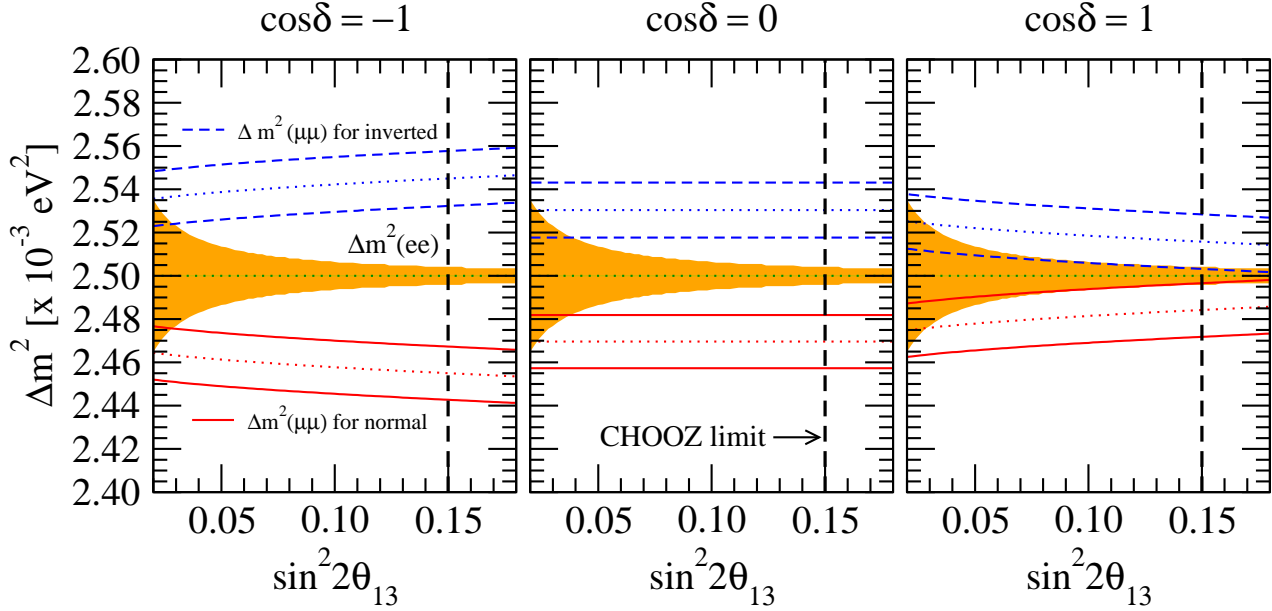


FIG. 1: Allowed regions for $\Delta m^2(\text{ee})$ (shaded by orange color) and $\Delta m^2(\mu\mu)$ (bands delimited by two solid and dashed curves) by measurement using the recoilless resonant $\bar{\nu}_e$ absorption reaction and the T2K II experiment, respectively, are plotted as functions of $\sin^2 2\theta_{13}$. The input value of $\Delta m^2(\text{ee}) = 2.5 \times 10^{-3} \text{eV}^2$ is assumed. The red solid (blue dashed) curve for $\Delta m^2(\mu\mu)$ denotes the case of normal (inverted) mass hierarchy. The left, the middle and the right panels are for the input values of $\delta = \pi$, $\delta = \pi/2$ or $3\pi/2$, and $\delta = 0$ or 2π , respectively.

A few remarks are in order:

- (1) The dependence of the fractional uncertainty of $\Delta m^2(\text{ee})$ which is proportional to $(\sin^2 2\theta_{13})^{-1}$ [22] is clearly visible in Fig. 1.
- (2) $\Delta m^2(\mu\mu)$ varies as a function of $\sin^2 2\theta_{13}$ because of the three-flavor effect in the disappearance probability $P(\nu_\mu \rightarrow \nu_\mu)$, see Eq. (4). Note, however, that the relative uncertainty with respect to its central value is independent of θ_{13} .
- (3) The three panels in Fig. 1, which correspond to different values of δ , indicate that the discriminating sensitivity of the mass hierarchy depends upon δ in an interesting way. The sensitivity is highest (lowest) at $\delta = \pi$ (0 or 2π), see Eq. (4).

To quantify the sensitivity region for the resolution of the mass hierarchy we define the probability distribution function $P_{\text{diff}}(\xi)$ of the difference $\xi \equiv \Delta m^2(\text{ee}) - \Delta m^2(\mu\mu)$. Then

the region of parameter which gives positive ξ at >90 , >95 and $>99\%$ CL are determined by the condition

$$\int_0^\infty d\xi P_{\text{diff}}(\xi) = 0.9, 0.95, 0.99. \quad (9)$$

Assuming that $\Delta m^2(\text{ee})$ and $\Delta m^2(\mu\mu)$ are Gaussian distributed³, $P_e(\Delta m^2(\text{ee}))$ and $P_\mu(\Delta m^2(\mu\mu))$, with the average values $\overline{\Delta m^2(\text{ee})}$ and $\overline{\Delta m^2(\mu\mu)}$ and widths σ_e and σ_μ , respectively, P_{diff} is also a Gaussian distribution with average value $\overline{\Delta m^2(\text{ee})} - \overline{\Delta m^2(\mu\mu)}$ and width $\sqrt{\sigma_e^2 + \sigma_\mu^2}$.

Using the precision for the determination of $\Delta m^2(\mu\mu)$ and $\Delta m^2(\text{ee})$ obtained in Secs. II and III, it is straightforward to determine the sensitivity regions. In Fig. 2 we present the sensitivity regions in the space spanned by $\sin^2 2\theta_{13}$ and δ at $> 90\%$ (green), $> 95\%$ (yellow), and $> 99\%$ (red) CL in which the mass hierarchy can be resolved by the method of comparing these two disappearance measurements.

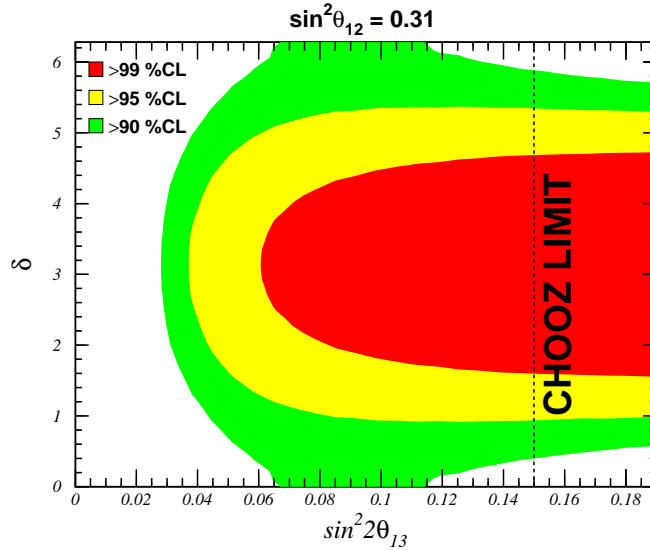


FIG. 2: Sensitivity regions in the $\sin^2 2\theta_{13} - \delta$ plane in which the mass hierarchy can be resolved at $>90\%$ (green), $> 95\%$ (yellow), and $>99\%$ (red) CL by the method of comparing the two disappearance measurements. The uncertainty on $\Delta m^2(\text{ee})$ is roughly given by $(0.3/\sin^2 2\theta_{13})\%$ under the assumed 0.2% systematic error and the uncertainty on $\Delta m^2(\mu\mu)$ is assumed to be 0.5% . Here the current best fit values $\sin^2 \theta_{12} = 0.31$, is used.

As anticipated in the remark after Fig. 1, the sensitivity depends significantly on the CP violating phase δ . It is highest at $\delta = \pi$, and lowest at $\delta = 0$ or 2π . In fact, this behavior is easy to understand from Eq. (4), given that $\cos 2\theta_{12}$ is positive definite, the highest (lowest) sensitivity is reached at $\cos \delta = -1$ ($+1$).

The strong dependence of the sensitivity on δ suggests that similar significant dependences exist also on other quantities, θ_{12} , $\Delta m_{21}^2/\Delta m^2(\alpha\alpha)$ and $\tan \theta_{23}$. In Fig. 3, the sensitivity regions for determining the mass hierarchy are plotted for two values of θ_{12} different from the

³ In good approximation, the χ^2 distribution of $\Delta m^2(\text{ee})$ is Gaussian as far as we exploit the setting discussed in [22].

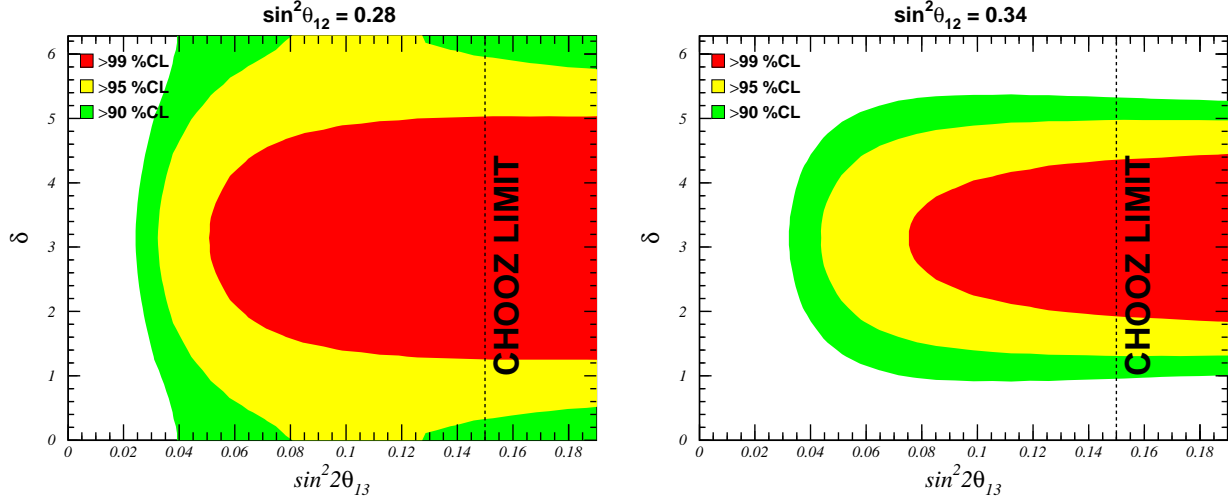


FIG. 3: Same as in Fig. 2 but for $\sin^2 \theta_{12} = 0.28$ (left panel) and 0.34 (right panel) which are allowed by the solar neutrino data and KamLAND at 2σ CL.

best fit one but allowed by the current data at 2σ CL. It is remarkable that the sensitivity regions depend so strongly on θ_{12} which corresponds to $\cos 2\theta_{12} = 0.38^{+0.061}_{-0.072}$. The smaller θ_{12} the better the sensitivity. At small θ_{13} the second term in the right-hand side of Eq. (4) is small, and the difference between the two Δm^2 is governed by the first term. The term is larger for smaller θ_{12} in the first octant, and hence the sensitivity is higher. Similar argument applies to $\Delta m_{21}^2/\Delta m^2(\alpha\alpha)$. If the true value of this ratio turns out to be larger (smaller) than the current best fit, it will be easier (more difficult) to determine the hierarchy.

We have presented the dependence of the sensitivity on θ_{12} because the significant improvement of the accuracy is not expected for this quantity, unless either precision measurement of low-energy solar neutrino flux is realized [23], or a dedicated reactor θ_{12} measurement is carried out [24]. On the contrary, the accuracy of Δm_{21}^2 determination will be improved to about 5% level by KamLAND alone as 10 times more statistics is accumulated by this experiment together with a better control of the systematic uncertainties [25]. Of course, there is no need to discuss the dependence on $\Delta m^2(\text{ee})$ and $\Delta m^2(\mu\mu)$ because they must be determined with high accuracies to realize the method. To determine $\tan \theta_{23}$ accurately we need to solve the θ_{23} octant degeneracy by either the atmospheric [26] or the reactor-accelerator combined methods [15].

Let us make some comments about the possibility to determine the CP phase δ . Unfortunately, it will not be possible to establish CP violation ($\sin \delta \neq 0$) by this method without even further improvement on the precision with which both Δm^2 's are measured. This can be understood by looking at Fig. 1. We can see that any point within the allowed region of $\Delta m^2(\mu\mu)$ delimited by the red solid (blue dashed) lines for $\cos \delta = 0$ (corresponding to $\delta = \pi/2$ or $3\pi/2$, i.e., CP violation) can also be within the allowed region either for $\cos \delta = 1$ or $\cos \delta = -1$ (corresponding to no CP violation). However, if CP is not violated, there is some possibility to distinguish $\delta = 0$ from $\delta = \pi$ since the two allowed regions for $\cos \delta = 1$ and $\cos \delta = -1$ have little overlap for $\sin^2 2\theta_{13} \gtrsim 0.05$. It is interesting to note that accelerator experiments like T2K II which will operate at the first oscillation maximum will have difficulty to distinguish the two points $\delta = 0$ and π [27].

Finally, let us also discuss the case where one of the assumption on the precision for Δm^2 is not valid. In the left panel of Fig. 4, we show the same information as in Fig. 2 but

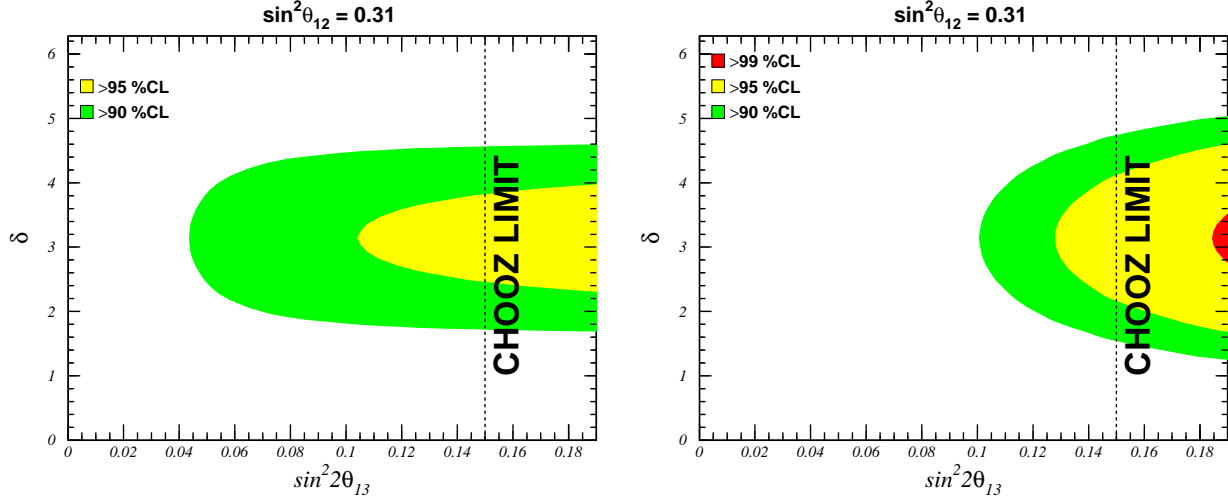


FIG. 4: Same as in Fig. 2 except: (left panel) the uncertainty on $\Delta m^2(\mu\mu)$ is increased by a factor of 2 to 1%, (right panel) systematic error for the $\Delta m^2(ee)$ determination was increased by a factor of 5 (from 0.2% to 1%) which implies that the uncertainty on $\Delta m^2(ee)$ is roughly described by $(1.2/\sin^2 2\theta_{13})\%$ with the uncertainty on $\Delta m^2(\mu\mu)$ returned to 0.5%.

assuming that the precision for $\Delta m^2(\mu\mu)$ is 1% (instead of 0.5%) but keeping the accuracy for $\Delta m^2(ee)$ the same as before. In this case the sensitivity is significantly reduced compared to the case presented in Fig. 2, but there is still significant parameter space where the mass hierarchy can be determined at $> 90\%$ CL. On the other hand, in the right panel of Fig. 4, we show the same information as in Fig. 2 but assuming the systematic uncertainty for the Mössbauer experiment is 1% (instead of 0.2%), which implies about the factor four larger uncertainty $\sim (1.2/\sin^2 2\theta_{13})\%$, on $\Delta m^2(ee)$, but keeping the accuracy for $\Delta m^2(\mu\mu)$ the same as before (0.5%). In this case, there is much less parameter space where we can determine the mass hierarchy.

V. CONCLUDING REMARKS

We have shown that if $\Delta m^2(\mu\mu)$ and $\Delta m^2(ee)$ can be measured, respectively, with 0.5% and $\sim 0.3 (\sin^2 2\theta_{13}/0.1)^{-1} \%$ accuracy, the neutrino mass hierarchy can be resolved down to $\sin^2 2\theta_{13} \sim 0.05$ for $0.3\pi \lesssim \delta \lesssim 1.7\pi$ at $>95\%$ CL for the current best fit values of the oscillation parameters. For slightly different values of the oscillation parameters, still allowed by data, the sensitivity can be even higher. Although it is not possible to determine the CP phase δ using these two measurements, it may be possible to distinguish the case $\delta = 0$ from $\delta = \pi$.

It should be stressed that high precision measurements of the muon and electron disappearance oscillation experiments considered in this paper have their own good physics motivation. If these two experiments will be realized, then as a special bonus, there is a possibility that the mass hierarchy can be determined, even though neither of these experiments alone can accomplish this task.

Existing technologies seem to allow us to realize the experiment such as the second phase of the T2K project, in which the determination of $\Delta m^2(\mu\mu)$ up to 2% precision looks quite feasible. How much one will be able to improve this precision depends essentially on how

much one can reduce the energy scale uncertainty. If it can be reduced to less than 1%, then it is worthwhile to consider the method studied in this paper.

The method discussed in this paper, “ Δm^2 -Disappearance” method, is most sensitive when δ is near π . Whereas long baseline experiments are most sensitive when $\delta = 3\pi/2$ ($\pi/2$) for the normal (inverted) mass hierarchy, [28, 29]. Thus, these two methods are complimentary in the CP violating phase variable, δ .

On the other hand, it is not yet clear if the oscillation experiment based on the Mössbauer enhanced resonant $\bar{\nu}_e$ absorption can be really performed. If it will be realized, the direct counting of ^3H atoms in the target is quite essential to carry out the measurement of $\Delta m^2(\text{ee})$ to the required accuracies.

To measure both $\Delta m^2(\text{ee})$ and $\Delta m^2(\mu\mu)$ to the accuracy less than 1% is challenging, but if it is possible, this will provide a direct and alternative way to determine the neutrino mass hierarchy.

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APPENDIX A: THE UNCERTAINTY ON Δm^2 IS BOUNDED FROM BELOW BY THE ABSOLUTE ENERGY SCALE UNCERTAINTY

We assume, as an idealistic situation, that the neutrino beam is monitored by a near detector with identical detection apparatus to a far detector for measuring modulation of the neutrino spectrum. Also, the neutrino source is assumed to be a point source for both the near and far detectors. With the energy scale uncertainty ΔE , the true and the reconstructed neutrino energies, E_{true} and E_{rec} , are related in both detectors by $E_{\text{rec}} = E_{\text{true}} + \Delta E = (1 + x)E_{\text{true}}$ where $x \equiv \Delta E/E_{\text{true}}$. For simplicity, we ignore all the other uncertainties except for the energy scale one and assume that the fractional uncertainty x is constant over the energy range of interest. In this case, it is clear that the number of events observed per unit of detector mass at the near and far detectors, N_{far} and N_{near} , respectively, are related to the neutrino disappearance probability P by

$$\begin{aligned} N_{\text{far}}(E_{\text{rec}}) &= \left(\frac{L_{\text{near}}}{L_{\text{far}}} \right)^2 N_{\text{near}}(E_{\text{rec}}) P \left(\frac{\Delta m_{\text{true}}^2 L_{\text{far}}}{E_{\text{true}}} \right) \\ &= \left(\frac{L_{\text{near}}}{L_{\text{far}}} \right)^2 N_{\text{near}}(E_{\text{rec}}) P \left(\frac{\Delta m_{\text{fit}}^2 L_{\text{far}}}{E_{\text{rec}}} \right), \end{aligned} \quad (\text{A1})$$

where L_{near} (L_{far}) is the near (far) baseline. This implies

$$\Delta m_{\text{fit}}^2 = \left(\frac{E_{\text{rec}}}{E_{\text{true}}} \right) \Delta m_{\text{true}}^2 = (1 + x) \Delta m_{\text{true}}^2. \quad (\text{A2})$$

So it is obvious that the percentage energy scale uncertainty sets a lower bound on the percentage uncertainty of the Δm^2 determination.

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